Subject Code: BAS103

PAPER ID-311443

Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2023-24
ENGINEERING MATHEMATICS-I

TIME: 3HRS **M.MARKS: 70**

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

Q no.	Question	Marks	CO
a.	Find the product and sum of the eigen values for $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.	2	1
b.	Find all symmetry in the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$.	2	2
c.	Calculate the error in R if $E = RI$ and possible errors in E and I are 30% and 20% respectively.	2	3
d.	Determine the value of $\Gamma\frac{1}{4}\,\Gamma\frac{3}{4}$.	2	4
e.	Prove that $B(p,q) = B(p+1,q) + B(p,q+1)$	2	4
f.	Prove that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational.	2	5
g.	Find a unit normal vector to the surface $xy^3z^2 = 4$ at the point $(-1,-1,2)$.	2	5

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

a.	Solve the system of homogenous equations: $x_1 + x_2 + x_3 + x_4 = 0,\ \ x_1 + 3x_2 + 2x_3 + 4x_4 = 0,$ $2x_1 + x_3 - x_4 = 0$	7	1
b.	If $u = y^2 e^{y/x} + x^2 \tan^{-1}\left(\frac{x}{y}\right)$, show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u.$	7	2
c.	Expand $f(x,y) = e^x \cos y$ about the point $\left(1, \frac{\pi}{4}\right)$ by Taylor's sereies.	7	3
d.	Evaluate the integral $\iint_D (y-x)\,dxdy$; by changing the variables, D: Region in xy-plane bounded by the lines $y - x = -3, y - x = 1, y + \frac{1}{3}x = \frac{7}{3}, y + \frac{1}{3}x = 5.$	7	4
e.	Find the directional derivative of $f(x,y,z) = e^{2x}\cos yz$ at $(0,0,0)$ in the direction of the tangent to the curve $x = a\sin\theta, y = a\cos\theta, \quad z = a\theta$ at $\theta = \pi/4$.	7	5

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Determine eigen vectors for the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.	7	1
b.	Determine A^{-1}, A^{-2} and A^{-3} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem.	7	1

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4. Attempt any *one* part of the following: **7 x 1 = 7**

a.	If $y = \cos(m \sin^{-1} x)$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$. Also find $(y_n)_0$.	7	2
b.	If $z = f(x, y), x = e^u + e^{-v}, y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$	7	2

5. Attempt any *one* part of the following: **7 x 1 = 7**

a.	If $u^3 + v^3 = x + y, u^2 + v^2 = x^3 + y^3$, then show $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2 - x^2}{2uv(u-v)}$.	7	3
b.	The pressure P at any point (x, y, z) in space is $P = 400\, xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method.	7	3

6. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Find the volume of the solid bounded by the coordinate planes and the surface $\left(\frac{x}{a}\right)^{1/2} + \left(\frac{y}{b}\right)^{1/2} + \left(\frac{z}{c}\right)^{1/2} = 1$.	7	4
b.	Prove that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$.	7	4

7. Attempt any *one* part of the following: **7 x 1 = 7**

a.	Applying Gauss Divergence theorem, evaluate $\iint_S [e^x dydz - ye^x dzdx + 3zdxdy]$, where S is the surface of the cylinder $x^2 + y^2 = c^2, 0 \leq z \leq h$.	7	5
b.	Prove that $\nabla^2 r^n = n(n + 1)r^{n-2}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$.	7	5