

**B. TECH.**  
**(SEM IV) THEORY EXAMINATION 2022-23**  
**MATHEMATICS V**

Time: 3 Hours

Total Marks: 100

**Note:** Attempt all Sections. If require any missing data; then choose suitably.

**SECTION A**

**1. Attempt all questions in brief. 2 x 10 = 20**

- (a) Find the Z-transform of  $n^2 e^{n\theta}$
- (b) Write the complex form of Fourier integral representation of a function.
- (c) The random variable X is said to follow the Normal distribution with mean 9 and standard deviation 3, find  $x^*$  such that  $P(X > x^*) = 0.16$ .
- (d) Check whether  $f(x) = \begin{cases} 0 & x < 0 \\ \alpha e^{-\alpha x}, \alpha > 0, & x \geq 0 \end{cases}$  is a Probability density function or not.
- (e) Write Lagrange's formula for inverse interpolation.
- (f) What is the rate of convergence for Newton-Raphson method.
- (g) What is F-test? Write the expression and application.
- (h) Write down the definition of the null hypothesis.
- (i) What is Statistical Quality Control (SQC)? Define in brief.
- (j) Define R chart with one application.

**SECTION B**

**2. Attempt any three of the following: 10x3=30**

- (a) Find the Fourier cosine transform of the following function  $f(x) = \frac{1}{1+x^2}$  and hence derive Fourier sine transform of  $\frac{x}{1+x^2}$
- (b) In a normal distribution, 12% of the items are under 30 and 85% items are under 60. Find the mean and standard deviation.
- (c) Determine  $f(x)$  as a polynomial in  $x$  for the following data:

<b>x:</b>	-4	-1	0	2	5
<b>f(x):</b>	1245	33	5	9	1335

Also estimate functional value if  $x = -2$ .

- (d) The annual rainfall in Lucknow city is normally distributed with mean 45 cm. The rainfall during the last five years are 48 cm, 42 cm, 40 cm, 44 cm and 43 cm. Can we conclude that the average rainfall during the last five years is less than the normal rainfall? Test at 5% level of

significance. [The tabulated value of  $t_{0.05} = 2.776$  and  $t_{0.1} = 2.132$  for 4 degree of freedom.]

- (e) What are the principles of experiment design? Discuss the completely randomized design in detail.

### SECTION C

**3. Attempt any one part of the following: 10x1=10**

- (a) Apply the Fourier transform to solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ ,  $x > 0, t > 0$   
if  $u(0, t) = 0$ ,  $u(x, 0) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$ , and  $u(x, t)$  is bounded.
- (b) Solve the following difference equation using Z-transform:  
 $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$  with  $u_0 = 0, u_1 = 0$ .

**4. Attempt any one part of the following: 10x1=10**

- (a) If  $X$  variable follow the Poisson distribution such that  
 $P(X = 2) = 9P(X = 4) + 90P(X = 6)$ . Find mean and variance of distribution.
- (b) For a continuous random variable  $X$  if  $f(x) = \frac{3}{4}(x^2 + 1)$ ,  $0 \leq x \leq 1$ . Then,  
(i) Find mean of random variable  $X$ .  
(ii) Find  $\alpha$  such that  $P(X \leq \alpha) = P(X > \alpha)$ .

**5. Attempt any one part of the following: 10x1=10**

- (a) From the following table, estimate the number of students who obtained marks between 40 & 45:

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

- (b) Find the root of the equation  $\cos x = xe^x$  using Regula-falsi method correct to 4 decimal places.

**6. Attempt any one part of the following: 10x1=10**

- (a) In an experiment on pea breeding the following frequency of seeds were obtained:

Red & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

Theory predicts the frequencies should be in the proportions 9:3: :3:1. Examine the correspondence between theory and experiment. Test at 5% level of significance. [The tabulated value of  $\chi^2_{0.05} = 7.815$  for 3 degree of freedom.]

- (b) In two independent sample of size 8 and 10, the sum of the square of deviations of the sample values from the respective means are 84.4 and 102.6. Test whether the differences of variances

of populations is segment or not. Use 5% level of significance. [ $F_{0.05}(7,9) = 3.29$ ].

**7. Attempt any one part of the following:**

**10x1=10**

- (a) Following is the data of defectives of 10 samples of size 100 each.

Sample No.	1	2	3	4	5	6	7	8	9	10
No. of Defectives	15	11	9	6	5	4	3	2	7	1

Construct p-chart and state whether the process is in statistical control or not.

- (b) The given table shows that the value of sample mean  $\bar{X}$  and the range R for 10 samples of size 5 each. Draw mean and range chart and also comment on the state of control of the process.

(Given  $A_2 = 0.58, D_3 = 0, D_4 = 2.115$ ).

Sample No.	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	45	46	48	52	53	37	51	46	47	38
R	4	5	6	7	4	5	7	6	6	4

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