B. TECH.
(SEM IV) THEORY EXAMINATION 2022-23
MATHEMATICS III

Time: 3 Hours   *Total Marks: 100*

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1.   **Attempt** *all* **questions in brief.**   2 x 10 = 20

   (a)   Evaluate $L[u_4(t) \sin \pi t]$, where $u_a(t)$ represents the unit step function.

   (b)   Describe the sufficient condition for the existence of Laplace transform of any function.

   (c)   If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, evaluate u_2. Here $U(z)$ represents the Z-transform of u_n.

   (d)   Write the complex form of Fourier integral representation of a function.

   (e)   Test whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 3 & 5 & 4 & 6 \end{pmatrix}$ is even permutation or odd permutation?

   (f)   State the Lagrange's theorem with reference to Group theory.

   (g)   In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics & Biology both and also the number of students who have taken Biology but not Mathematics.

   (h)   If function $f: R \to R$ is defined as $f(x) = x^2$, find $f^{-1}(4)$ and $f^{-1}(-9)$.

   (i)   Define Lattice and Distributive lattice with examples.

   (j)   Prove the following absorption laws in Boolean algebra:

   $(i)\ a + (a.b) = a \quad (ii)\ a.(a+b) = a$

SECTION B

2.   **Attempt any** *three* **of the following:**   10x3=30

   (a)   Apply the Convolution theorem to find the inverse Laplace transform $L^{-1}[\frac{s^2}{(s^2+a^2)(s^2+b^2)}]$

   (b)   Apply the Fourier transform to solve the Heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$,

   if $u(0,t) = 0$, $u(x,0) = e^{-x}, x > 0$ and $u(x,t)$ is bounded for $x > 0, t > 0$.

   (c)   Check the validity of the following argument:

   If 2 is an odd number then 4 does not divide 10. Either 11 is not a prime number or 4 divides 10.

   But 11 is a prime number, therefore 12 is an even number.

   (d)   Solve the following Recurrence relation:

   $a_n - 7a_{n-1} + 10a_{n-2} = n + 2$ given $a_0 = 1, a_1 = 1$

   (e)   Convert the Boolean expression $[(xy' + xz)' + x']$ into Disjunctive normal form and Conjunctive normal form.

SECTION C

3. **Attempt any *one* part of the following:** 10x1=10

(a) Solve the following differential equation using Laplace transform:

$$[D^3 - 3D^2 + 3D - 1]y = t^2 e^t, \text{ given that } y(0) = 1, y'(0) = 0, \ y''(0) = -2.$$

(b) Find the Laplace transform of the following function:

$$f(t) = \begin{cases} 0, & 0 \leq t \leq 1 \\ t-1, & 1 \leq t \leq 2 \\ 3-t, & 2 \leq t \leq 3 \\ 0, & t > 3 \end{cases}$$

4. **Attempt any *one* part of the following:** 10x1=10

(a) Find the Fourier transform of the following function:

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence find } \int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx$$

(b) Solve the following difference equation using Z-transform:

$$u_{n+2} - 5u_{n+1} + 6u_n = 2^n \text{ with } u_0 = 0, u_1 = 1.$$

5. **Attempt any *one* part of the following:** 10x1=10

(a) Prove that the set of all positive rational numbers Q^+ forms an abelian group under the

operation ' $*$ ', defined as $a * b = \frac{ab}{2}$.

(b) Test whether the statement $p \Rightarrow (\sim q \lor r)$ is a tautology or a contradiction or a contingency.

Also, test whether $p \Rightarrow (\sim q \lor r) \equiv (p \Rightarrow \sim q) \lor (p \Rightarrow r)$.

6. **Attempt any *one* part of the following:** 10x1=10

(a) Show that $n^2 > (2n+1)$ for $n \geq 3$ by Mathematical Induction.

(b) Let R be a relation in a set of integers Z, defined by

$R = \{(x,y): x - y \text{ is divisible by } 6, \ x \in Z, y \in Z\}$. Justify that R is an Equivalence relation or

Partially ordered relation in Z.

7. **Attempt any *one* part of the following:** 10x1=10

(a) Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be any set. The relation ' $\leqslant$ ' in set A is defined as $x \leqslant y$

when x divides y.

Is the set $(A, \leqslant)$ partially ordered set (poset)? If yes, draw the Hasse diagram for this poset and

hence list all the maximal, minimal, greatest and least elements of the poset.

(b) Draw a switching circuit for the following Boolean expression:

$$[a + (b+c).a'].b' + c.[b.(a+c') + a.(b'+c)]$$

Also draw a logic gate circuit for the minimized form of above Boolean expression.